Exercise 13

Consider the point (x, y) lying on the graph of the line $2x + 4y = 5$. Let L be the distance from the point (x, y) to the origin $(0, 0)$. Write L as a function of x.

Solution

The distance from (x, y) to $(0, 0)$ is given by

$$\begin{aligned} L &= \sqrt{(0-x)^2 + (0-y)^2} \\ &= \sqrt{(-x)^2 + (-y)^2} \\ &= \sqrt{x^2 + y^2}. \end{aligned} \tag{1}$$

Solve the given equation for y.

$$\begin{aligned} 2x + 4y &= 5 \\ 4y &= 5 - 2x \\ y &= \frac{5}{4} - \frac{1}{2}x \end{aligned}$$

Therefore, equation (1) becomes

$$\begin{aligned} L &= \sqrt{x^2 + \left(\frac{5}{4} - \frac{1}{2}x\right)^2} \\ &= \sqrt{x^2 + \left[\frac{25}{16} - 2\left(\frac{5}{4}\right)\left(\frac{1}{2}\right)x + \frac{1}{4}x^2\right]} \\ &= \sqrt{x^2 + \frac{25}{16} - \frac{5}{4}x + \frac{1}{4}x^2} \\ &= \sqrt{\frac{5}{4}x^2 - \frac{5}{4}x + \frac{25}{16}} \\ &= \sqrt{\frac{5}{16}(4x^2 - 4x + 5)} \\ &= \frac{\sqrt{5}}{4}\sqrt{4x^2 - 4x + 5}. \end{aligned}$$